

A Modification to the Enhanced Correction Factor Technique to Correlate With Experimental Data

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The current paper presents an extension to the Enhanced Correction Factor Technique for correcting unsteady lifting surface pressure coefficients used in flutter and dynamic aeroelastic loads analyses to produce realistic pressure distributions and correlate more closely to experimental unsteady aerodynamic pressures and wind tunnel flutter test results.

I. Introduction

Modern industrial aeroelastic analyses for transport category aircraft encompass a large number of conditions for both dynamic aeroelastic loads and flutter analyses. Although great advances have been made in the field of computational fluid dynamics (CFD), particularly for unsteady aerodynamic forces, the sheer number of cases required to be analyzed for a certification project dictates that a simpler, more expedient method be used.

Currently, industry mainly relies on linear lifting surface methods such as the Doublet Lattice Method (DLM)¹ or the ZAERO method to model the unsteady aerodynamics of lifting surfaces and bodies, due to reduced cost and relatively good representation of the lags between the surface motion and the aerodynamic forces. However as lifting surface methods are based on potential theory, they are not capable of capturing complex three dimensional, compressible or viscous effects directly. To overcome this limitation, standard industry practice is to correct the absolute value of the aerodynamic lifting pressures based on steady aerodynamics and rely on the as calculated phase lags. This combination is possibly the cheapest transonic unsteady aerodynamic method in terms of computing requirements. This makes possible the running of thousands of flight cases for both flutter and dynamic flight loads (such as discrete gust and continuous turbulence) and at least approximates complex flow effects.

Previous efforts have been undertaken to correct the DLM for given surface deformations²⁻³, typically the structural eigenmodes, with unsteady CFD runs to include the representativeness of the unsteady part of the aerodynamics and keep the cost at reasonable levels. A primary drawback of this approach is that the corrections are a function of both the mass and stiffness distributions of the aircraft.

Jadic, Hartley and Giri⁴ presented the Enhanced Correction Factor Technique (ECFT) methodology to correct the DLM forces by using given aerodynamic modes as well as arbitrary geometric modes. This paper presents further advancement of their work.

The current effort, undertaken as part of Gulfstream's Empenage Flutter Model (EFM) flutter test correlation⁵⁻⁶, shows that the original ECFT approach is very conservative and does not reproduce the physical behavior of the unsteady aerodynamic forces. Given the fact that during Gulfstream's Rigid Horizontal Tail Model wind tunnel test⁷ only the aerodynamic forces due to oscillation of the rigid pitch were measured, a different approach in which the DLM correction is formed as a diagonal matrix has been used. The resulting flutter speeds with this correction are much closer to the experiment, indicating that some aspect was not adequate in the original ECFT⁴ approach.

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II. Full ECFT

The first step needed to generate aerodynamic corrections is to have a set of experimental, or CFD generated, lifting pressure coefficients (ΔC_p 's) on each DLM panel due to different aerodynamic states.

The original ECFT⁴ used trigonometric series expansions as a basis to generate the different aerodynamic states. This, however, means that either new CFD runs are required or the existing runs have to be somehow assimilated to the series expansion, adding another layer of uncertainty.

Gultstream's approach uses the null subspace of the existing (because they are generated as part of other requirements in the loads analysis cycle) aerodynamic states. These are typically flight dynamics states such as angle of attack, angle of sideslip, control surface deflections, tail incidence, etc. Alternatively, they could also be any set of states, for example, different modal shapes.

Stacking the cp 's column-wise would lead to

$$\underbrace{[\Delta C_p]}_{N_j \times N_m}$$

where N_j is the size of the j -set (aerodynamic degrees of freedom) and N_m is the number of aerodynamic modes or states. For each of the aerodynamic modes we have a set of downwashes on the j -set, so, equivalently we would have

$$\underbrace{[w_j]}_{N_j \times N_m}$$

The fundamental idea behind the generation of the aerodynamic correction is to solve the system of equations

$$\underbrace{[W_{jj}]}_{N_j \times N_j} \underbrace{[A_{jj}^{-1}]}_{N_j \times N_j} \underbrace{[w_j]}_{N_j \times N_m} = \underbrace{[\Delta C_p]}_{N_j \times N_m}$$

Unfortunately this system is indeterminate, and therefore does not have a unique solution. Making use of the null subspace of $[w_j]$ a matrix of rank N_j is obtained:

$$\underbrace{[\Omega_j]}_{N_j \times N_j} = \begin{bmatrix} \underbrace{[w_j]}_{N_j \times N_m} & \vdots & \underbrace{[null(w_j)]}_{N_j \times (N_j - N_m)} \end{bmatrix}$$

Following the approach in the original ECFT paper⁴, the lifting pressures, $[\Delta C_p]$, are complemented with the pressures produced by the complimentary null subspace:

$$\underbrace{[P_j]}_{N_j \times N_j} = \begin{bmatrix} \underbrace{[\Delta C_p]}_{N_j \times N_m} & \vdots & \underbrace{[A_{jj}^{-1}] \cdot [null(w_j)]}_{N_j \times (N_j - N_m)} \end{bmatrix}$$

So, the final system of equations is

$$\underbrace{[W_{jj}]}_{N_j \times N_j} \underbrace{[A_{jj}^{-1}]}_{N_j \times N_j} \underbrace{[\Omega_j]}_{N_j \times N_j} = \underbrace{[P_j]}_{N_j \times N_j}$$

This system is determinate and has a unique solution.

It must be noted, however, that the means to close the system of equations is completely mathematical and not related to physics. It is only one of the infinite solutions available. The problem with this approach is that for any downwash in the null subspace matrix the aerodynamic output would be exactly the uncorrected DLM. For any other combination it would be a mix between corrected and uncorrected DLM.

III. Diagonal and Modified ECFT

Another way to obtain a unique solution is to assume that $[W_{jj}]$ is diagonal, such that $[w_j]$ is a vector corresponding to only one aerodynamic state or a combination of states (for example sideslip for vertical panels and angle of attack for the remaining panels).

$$\underbrace{\begin{bmatrix} W_{jj} & 0 \\ 0 & \ddots \end{bmatrix}}_{N_j \times N_j} \underbrace{[A_{jj}^{-1}]}_{N_j \times N_j} \underbrace{[w_j]}_{N_j \times 1} = \underbrace{[\Delta C_p]}_{N_j \times 1}$$

This approach has a more physical meaning, but does not make use of all the available information and does not match the CFD for our original set of modes.

As explained before for the original approach, the system is indeterminate and does not have a unique solution. Transposing the matrices to put them in $AX = B$ format, we get:

$$\underbrace{[w_j]^T}_{N_m \times N_j} \underbrace{[A_{jj}^{-1}]^T}_{N_j \times N_j} \underbrace{[W_{jj}]^T}_{N_j \times N_j} = \underbrace{[\Delta C_p]^T}_{N_m \times N_j}$$

It is known that if $[A^+]$ is the Moore-Penrose pseudo-inverse of $[A]$, any solution to the system of equations is generated by:

$$[X] = [A^+][B] + ([I] - [A^+][A])[Y]$$

Where $[Y]$ is an arbitrary matrix.

Pre-multiplying by $[I]$ we can search for the matrix $[Y]$ which provides the same diagonal $[W_{jj}]$ as the DIAGONAL ECFT method, but still gives the corrected aerodynamics for all the inputs states. In this way $[W_{jj}]$ is diagonally dominant, which has more physical sense, but still accurate for the remaining aerodynamic states.

$$\begin{aligned} [Y] &= ([I] - [I][A^+][A])^{-1} ([\text{Diag}_{W_{jj}}] - [I][A^+][B]) \\ [W_{jj}]^T &= [A^+][B] + ([I] - [A^+][A])[Y] \end{aligned}$$

IV. Expanding Aerodynamic Degrees of Freedom

The process outlined above calculates $[W_{jj}]$, but for some instances $[W_{kk}]$ on the k-set might be needed, for example as an input to MSC NASTRAN. Unfortunately, the transformation from $[W_{jj}]$ is not unique, as $[W_{kk}]$ pre-multiplies $[S_{kj}]$ in the generation of DLM forces and moments and $[W_{jj}]$ post-multiplies, as can be seen below:

$$[F_k] = q[W_{kk}][S_{kj}][A_{jj}^{-1}][w_j]$$

or

$$[F_k] = q[S_{kj}][W_{jj}][A_{jj}^{-1}][w_j]$$

For the transformation to be accurate, it should be shown that:

$$\underbrace{[W_{kk}]}_{N_k \times N_k} \underbrace{[S_{kj}]}_{N_k \times N_j} = \underbrace{[S_{kj}]}_{N_k \times N_j} \underbrace{[W_{jj}]}_{N_j \times N_j}$$

which is again an indeterminate system.

A physical way to solve the system, in the case of an aerodynamic model comprising only of lifting surfaces, is to assume that the correction for the moment does not multiply the normal forces and vice-versa:

$$([S_{kj}][W_{jj}])_{i,j} = [S_{kj}]_{2i-1,j} [W_{jj}]_{i,j}$$

$$([S_{kj}][W_{jj}])_{i+1,j} = [S_{kj}]_{2i,j} [W_{jj}]_{i,j}$$

$$([W_{kk}][S_{kj}])_{i,j} = [W_{kk}]_{i,2j-1}[S_{kj}]_{2j-1,j}$$

$$([W_{kk}][S_{kj}])_{i+1,j} = [W_{kk}]_{i+1,2j}[S_{kj}]_{2j,j}$$

This closes the system and allows to obtain $[W_{kk}]$.

V. Experimental Data Acquisition

For the past several years, Gulfstream has conceived and executed a wind tunnel test campaign at NASA Langley Research Center's specialized aeroelastic testing wind tunnel known as the Transonic Dynamics Tunnel (TDT). The data acquired during this campaign is being currently used to reduce the unsteady aerodynamics uncertainty at transonic speeds and validate the current state of the art methodology. This is especially important for flutter mechanisms that involve coupling of higher order primary surface modes with control surface rotation at high transonic Mach numbers and high reduced frequencies

The wind tunnel test campaign consisted of a two phase approach. The empennage configuration was chosen as it provided a target flutter mechanism that involved coupled main surface and control surface.

The first phase was completed in September 2011 and consisted of obtaining unsteady pressure data for a rigid horizontal tail model (RHTM) at a wide range of pitch oscillation frequencies and Mach numbers.

The second phase was completed in January 2013, and consisted of a flexible empennage model (EFM), designed to flutter in the TDT operating envelope. Two flutter mechanisms were found and their boundaries explored.

The first mechanism encountered during testing was an unexpected 10 Hz instability single degree of freedom instability of the first symmetric horizontal tail bending, which may have been caused by a shock wave emanating from the bottom fairing, impinging on the horizontal stabilizer. When the flow Mach number is between 0.85 – 0.92 the shock moves to the critical location on the underside of the HT and triggers the 10 Hz mode.

The second mechanism measured was the targeted 40-50 Hz coupled horizontal stabilizer torsion and elevator rotation. Two dynamic pressure points very close to the flutter envelope, but safe enough to maintain model integrity, were measured (see Figure 1) . The first one was for $M=0.833$ at a dynamic pressure of 168 psf and the second for $M=0.78$ at a dynamic pressure of 208 psf.

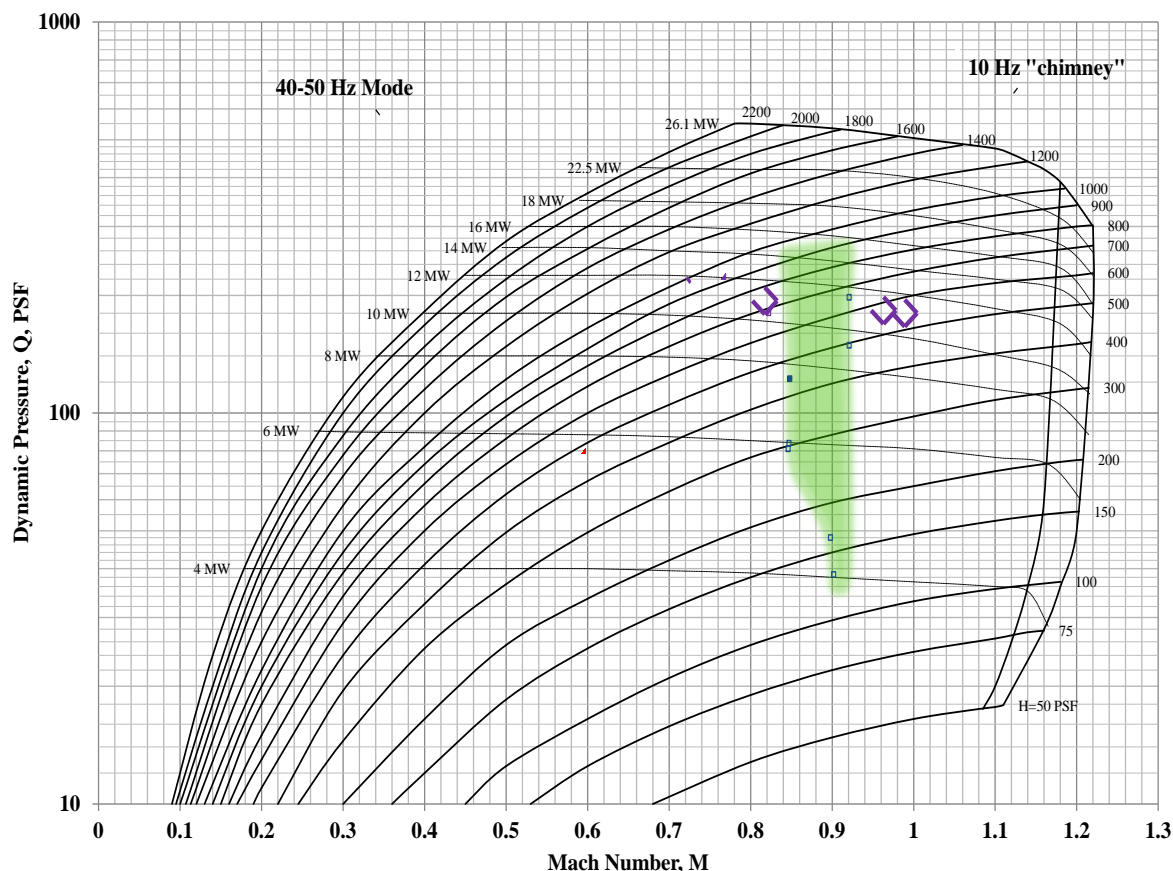


Figure 1 NASA LaRC TDT Operating Envelope

VI. Steady Wind Tunnel Data Correction ($M=0.8$)

The steady incremental pressure data obtained from the RHTM wind tunnel campaign steady pitching data has been mapped and used to generate aerodynamic correction factors using both the full ECFT and diagonal methods outlined in this paper, for $M=0.8$. Then, the corrected unsteady DLM data is compared with the measured unsteady data to check the frequency validity of the steady correction factors as shown in Figures 2-8.

As it can be seen, the correction is exact for the steady comparison, and starts diverging around 20 Hz (reduced frequency $=0.50$). This sets an upper limit for the steady correction validity in this case.

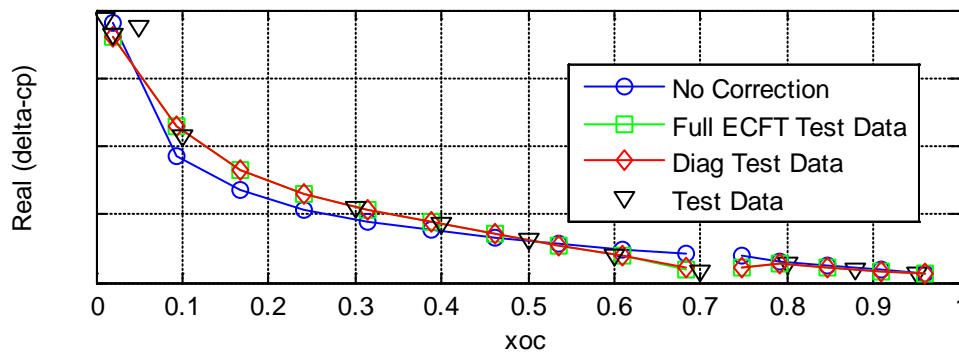


Figure 2. Rigid Alpha Mode Correction Comparison. 50% Span, $M=0.8$, Freq. $=0$ Hz

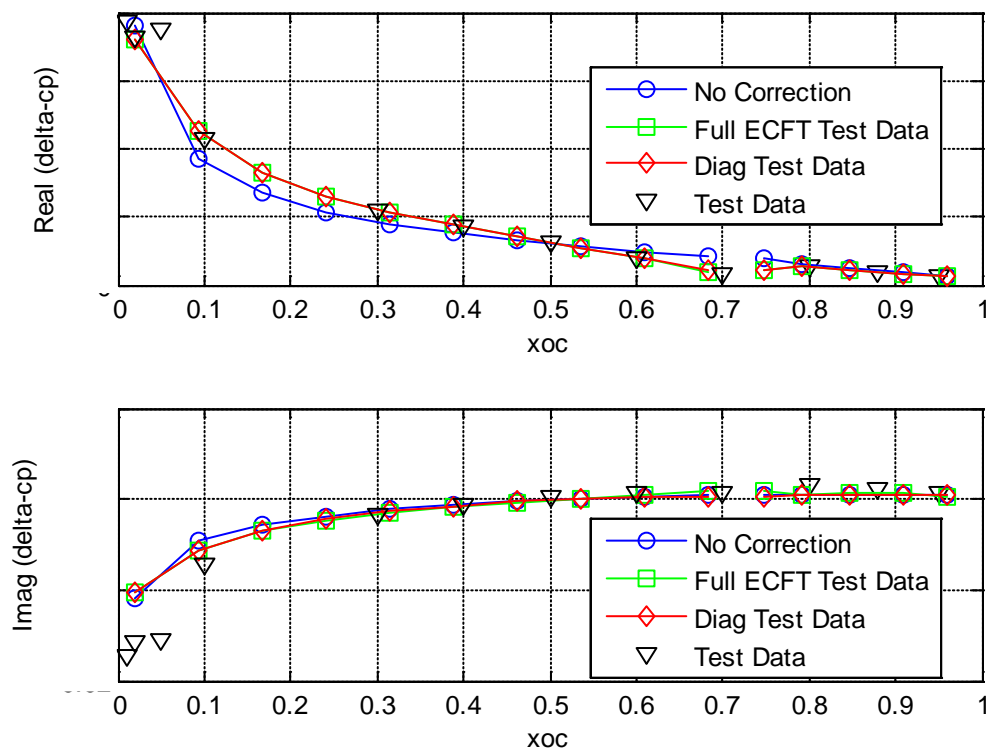


Figure 3. Rigid Alpha Mode Correction Comparison. 50% Span, $M=0.8$, $\text{Freq.}=1 \text{ Hz}$

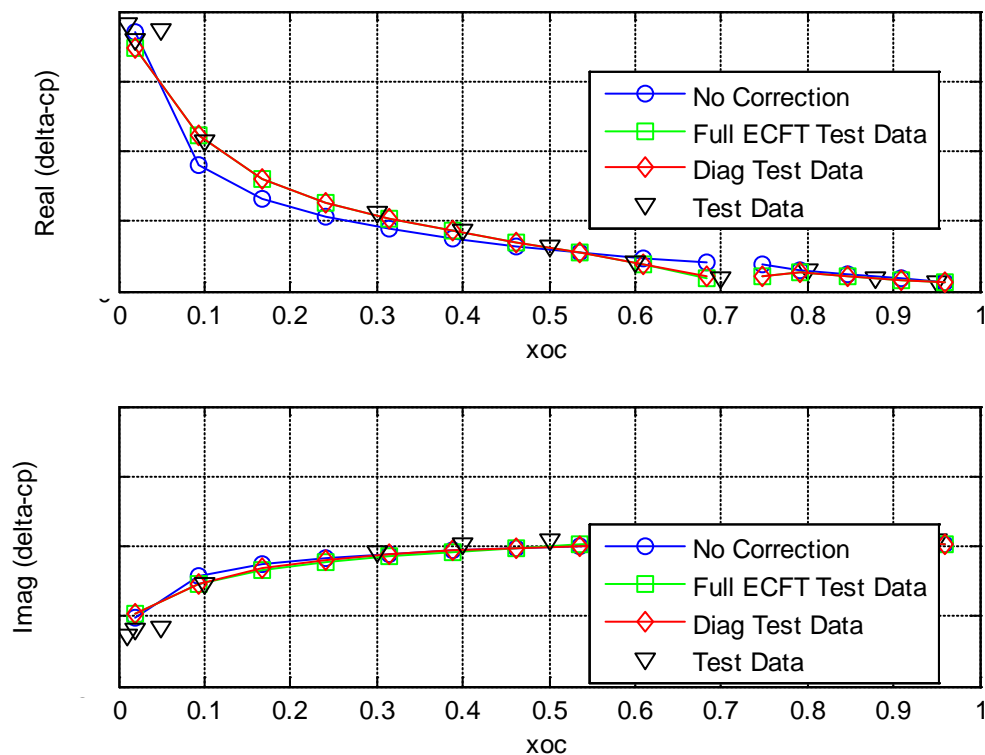


Figure 4. Rigid Alpha Mode Correction Comparison. 50% Span, $M=0.8$, $\text{Freq.}=5 \text{ Hz}$

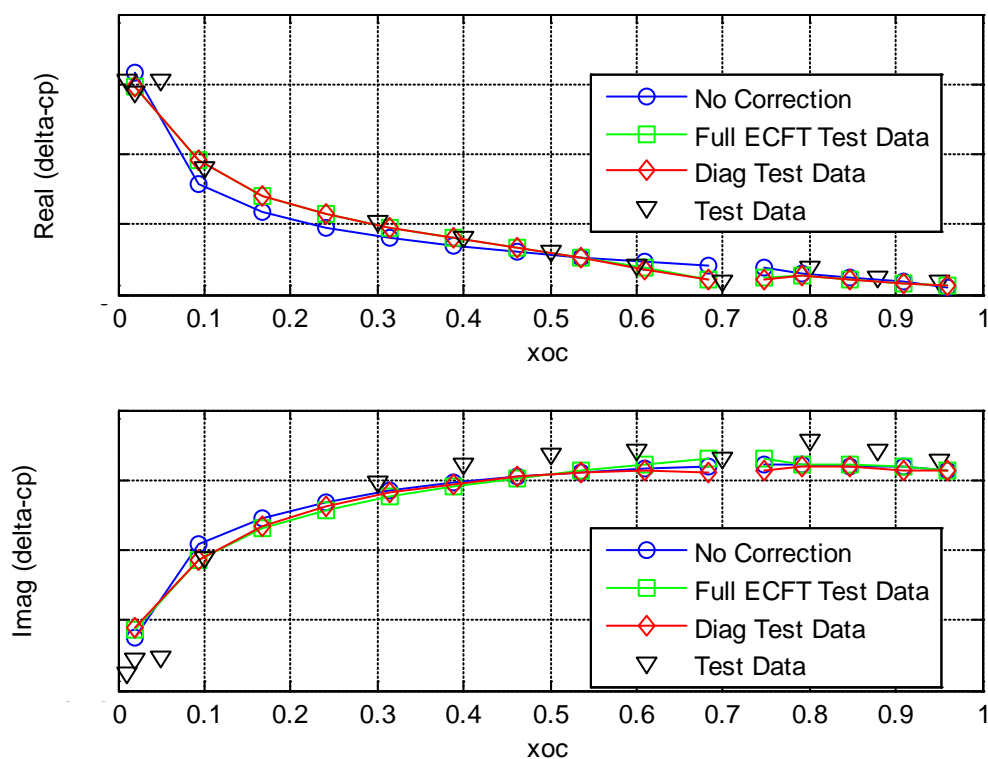


Figure 5. Rigid Alpha Mode Correction Comparison. 50% Span, $M = 0.8$, Freq. = 15 Hz

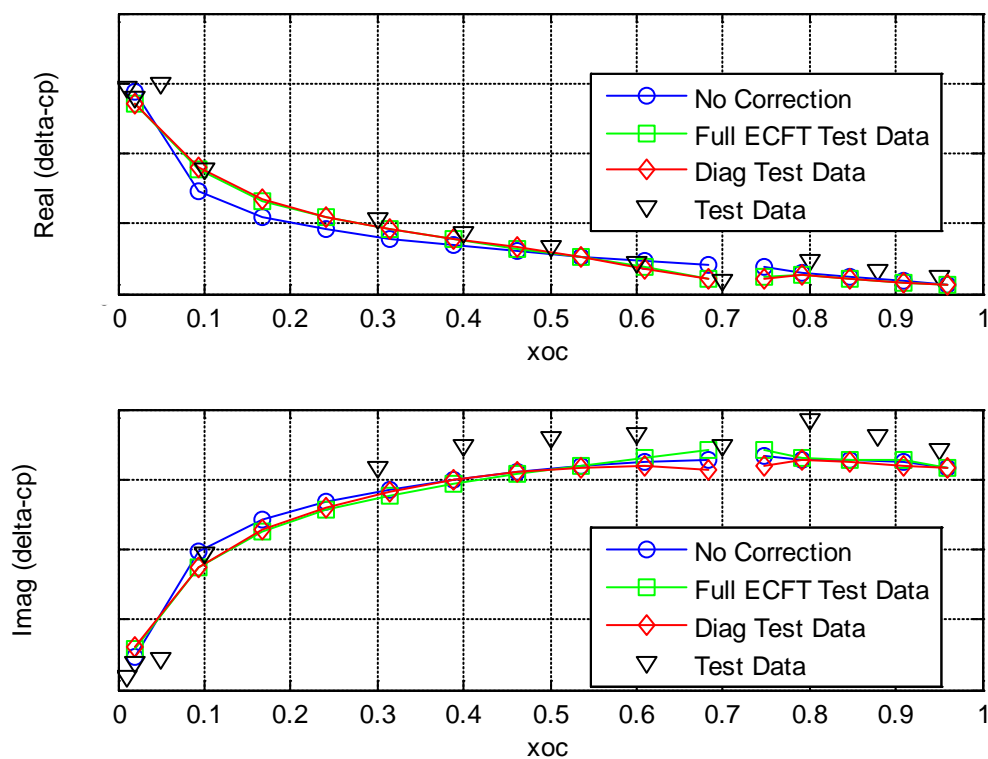


Figure 6. Rigid Alpha Mode Correction Comparison. 50% Span, $M = 0.8$, Freq. = 20 Hz

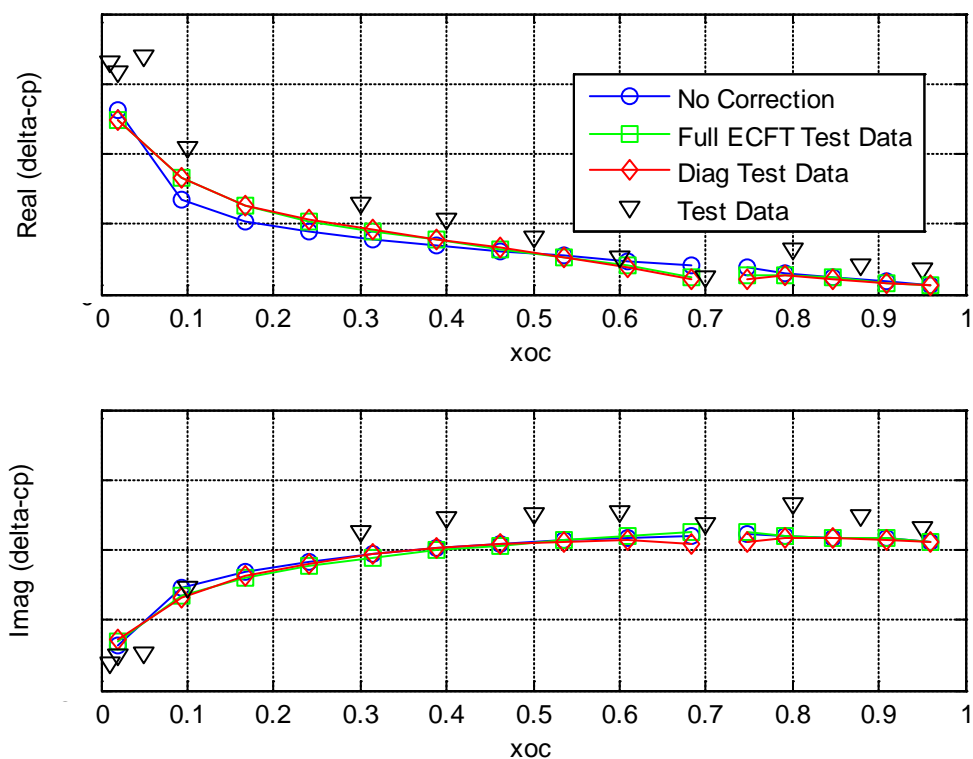


Figure 7. Rigid Alpha Mode Correction Comparison. 50% Span, $M = 0.8$, Freq. = 25 Hz

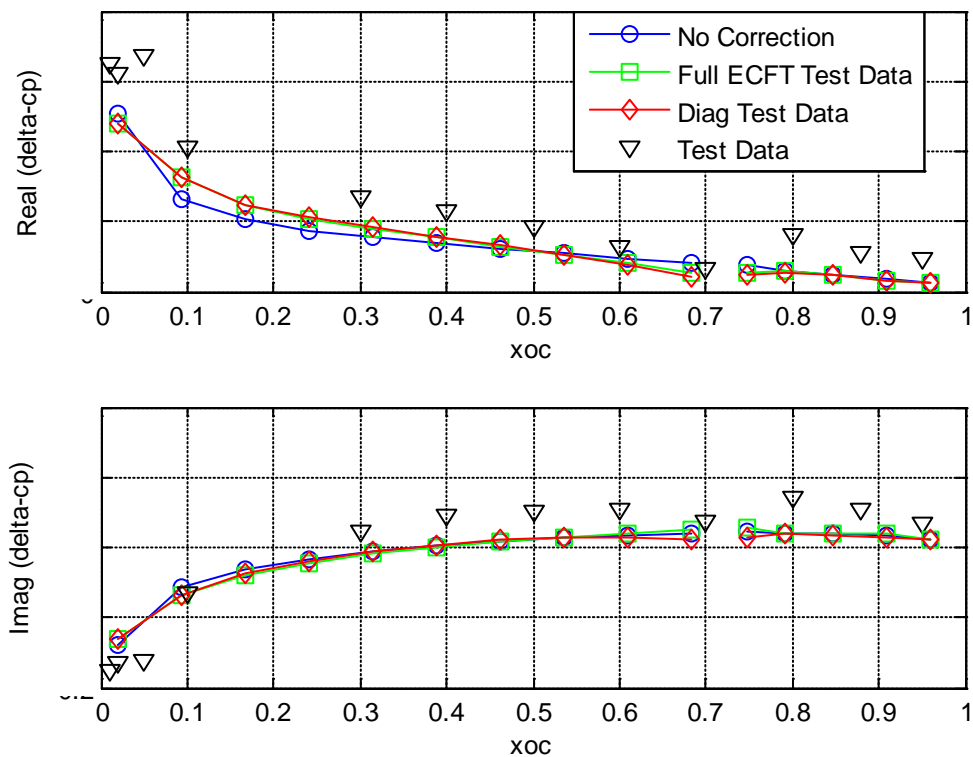


Figure 8. Rigid Alpha Mode Correction Comparison. 50% Span, $M = 0.8$, Freq. = 27 Hz

VII. Unsteady Data Correction at M=0.8

In an attempt to check the effect of the unsteady pitch correction, reduced frequency dependent corrections have been generated. Since the target flutter reduced frequency is higher than the acquired wind tunnel pitching test data, NASA Langley's aeroelastic RANS solver, FUN3D⁸ has been used. FUN3D has been compared against experimental data for the reduced set of measured pitching frequencies, and subsequently used to generate the frequency dependent correction matrices. The agreement between FUN3D and the test data is excellent (see figures 9-13), and thus the validity of the approach is confirmed.

Once the correction matrices are generated, the resultant DLM pressures are compared against the original FUN3D and against the previous steady correction(see figures 14-22). Once more the correction is exact, and is significantly different from the pressures that would result from the steady correction.

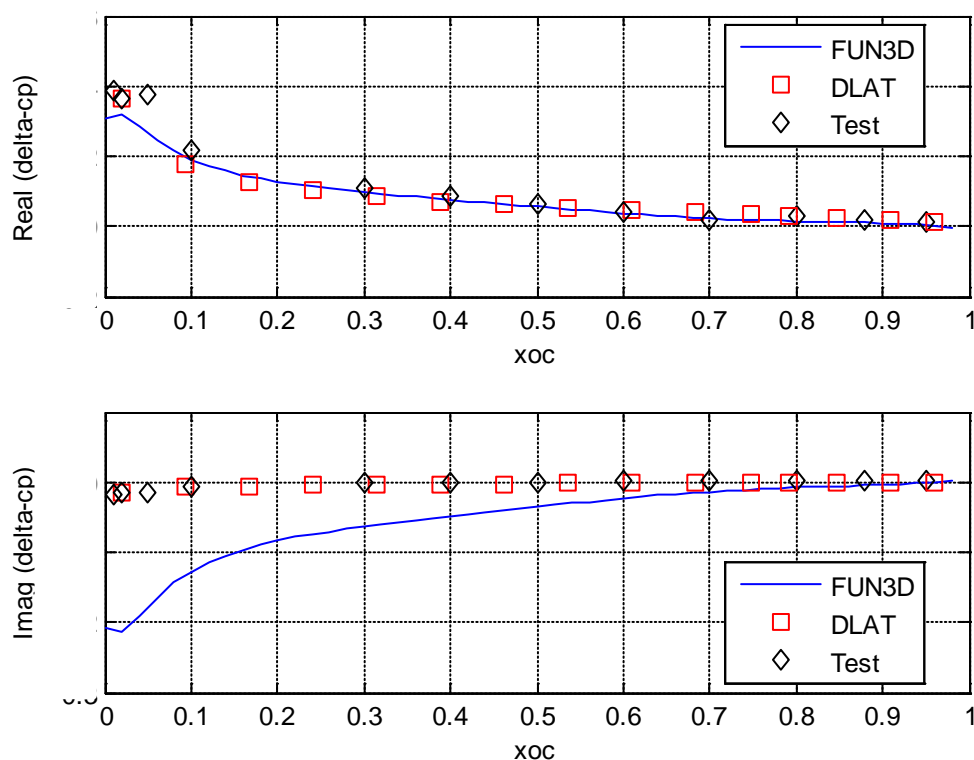


Figure 9. Rigid Alpha Comparison FUN3D vs DLAT vs Test. 50% Span, M =0.8, Freq.=1 Hz

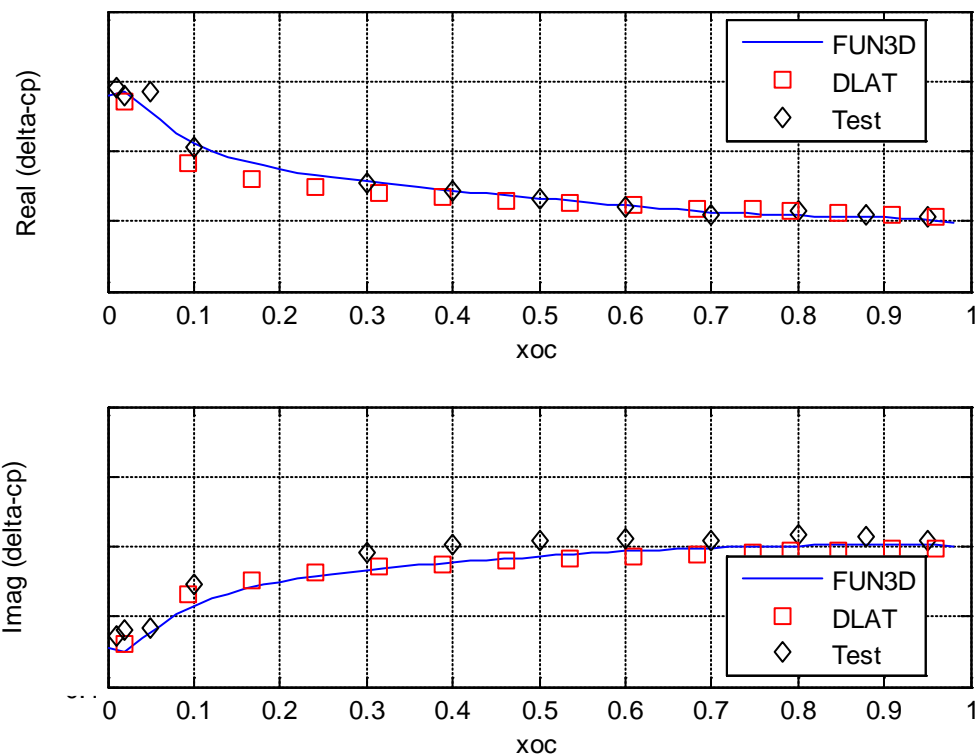


Figure 10. Rigid Alpha Comparison FUN3D vs DLAT vs Test. 50% Span, $M=0.8$, $\text{Freq.}=5 \text{ Hz}$

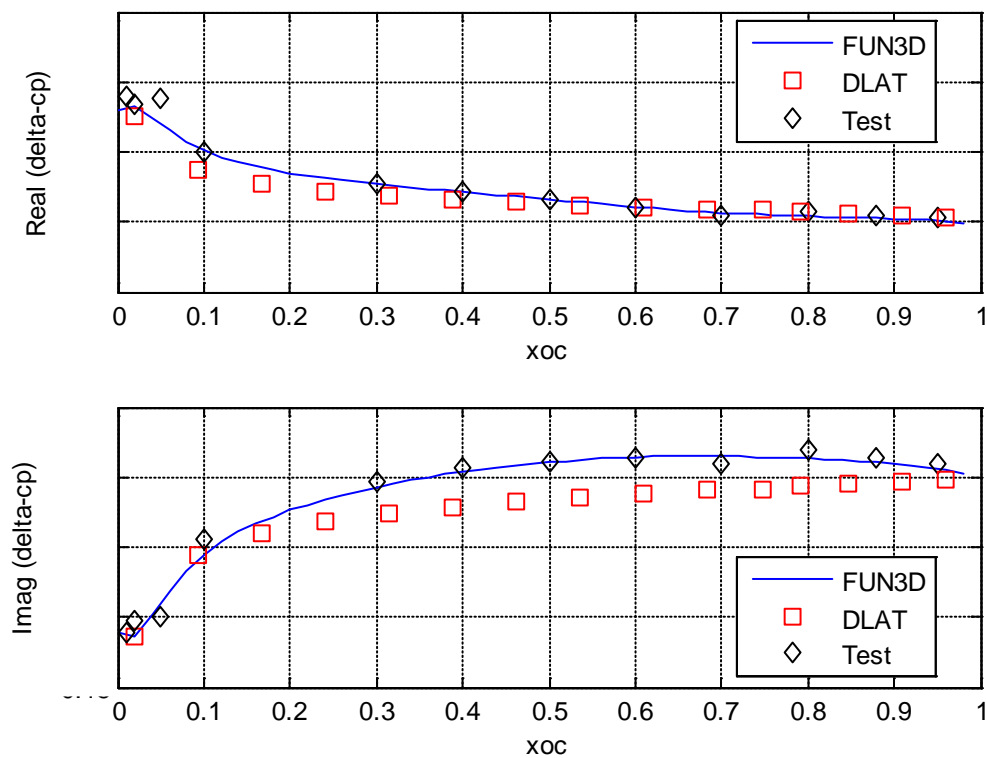


Figure 11. Rigid Alpha Comparison FUN3D vs DLAT vs Test. 50% Span, $M=0.8$, $\text{Freq.}=10 \text{ Hz}$

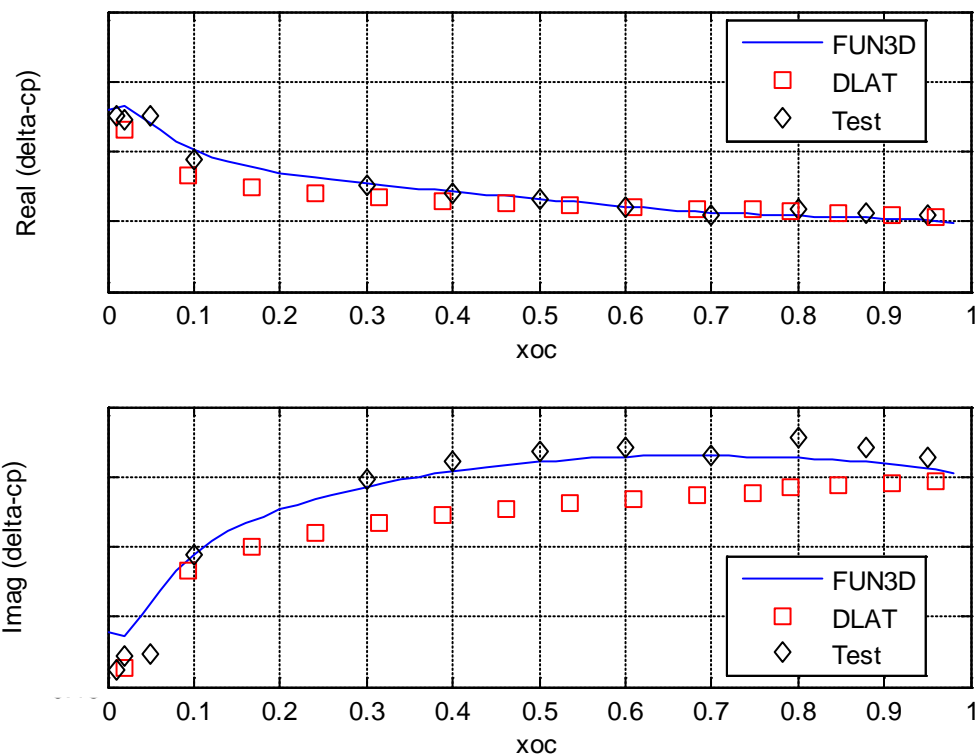


Figure 12. Rigid Alpha Comparison FUN3D vs DLAT vs Test. 50% Span, M =0.8, Freq.=15 Hz

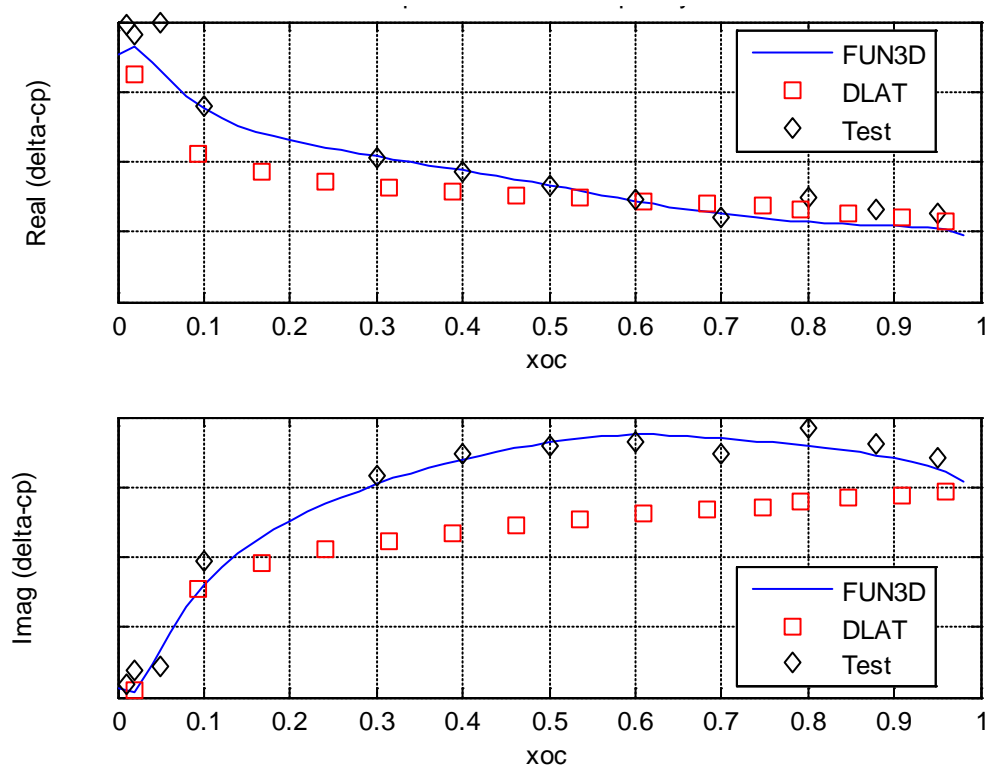


Figure 13. Rigid Alpha Comparison FUN3D vs DLAT vs Test. 50% Span, M =0.8, Freq.=20 Hz

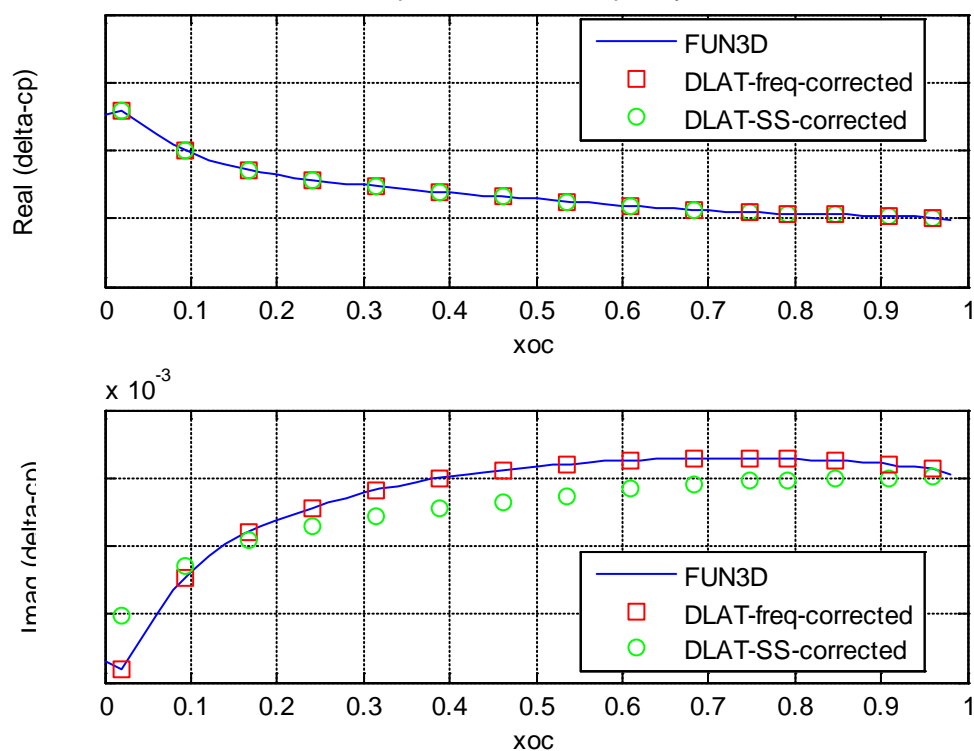


Figure 14. FUN3D vs Steady and Freq Correction. 50% Span, M =0.8, Freq.=1 Hz

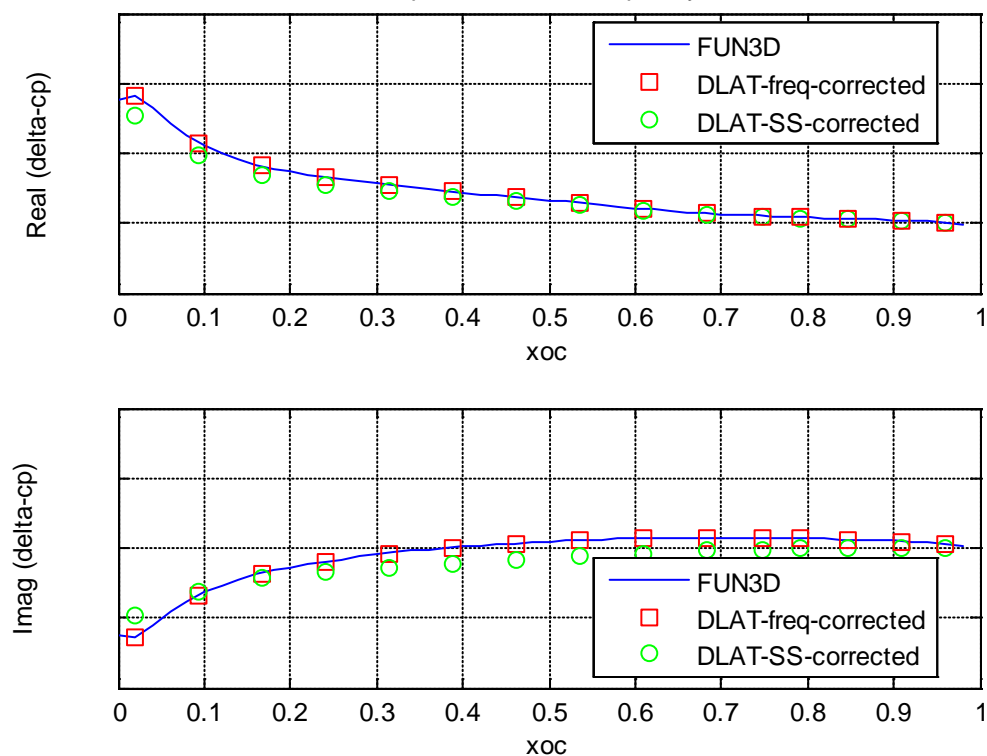


Figure 15. FUN3D vs Steady and Freq Correction. 50% Span, M =0.8, Freq.=5 Hz

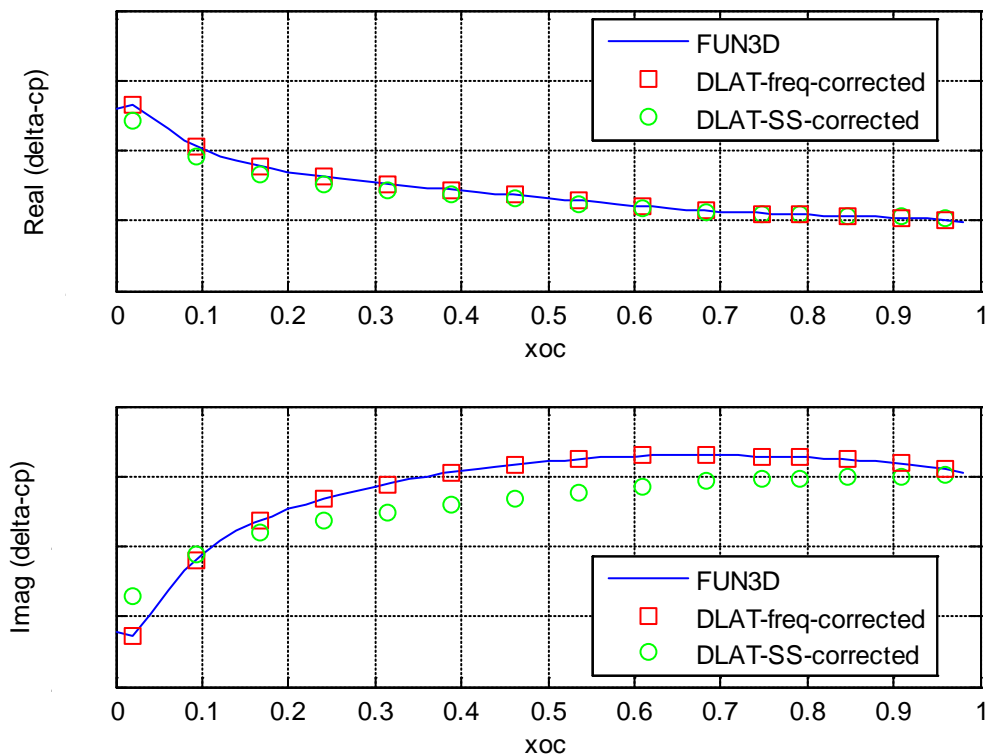


Figure 16. FUN3D vs Steady and Freq Correction. 50% Span, $M = 0.8$, $\text{Freq.} = 10 \text{ Hz}$

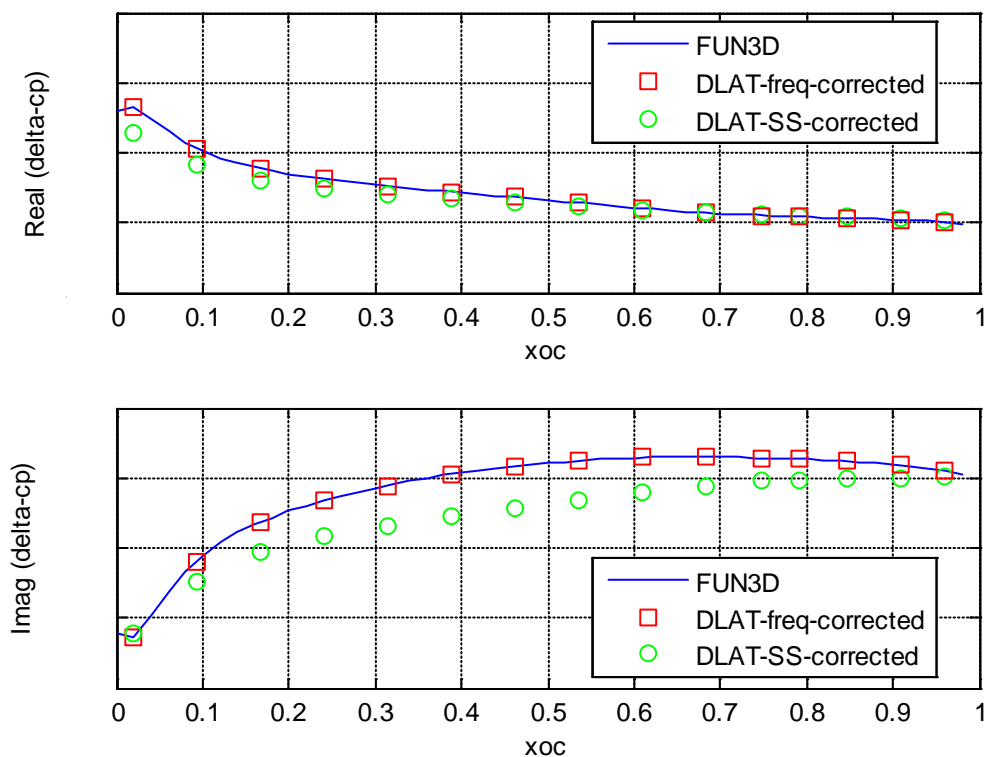


Figure 17. FUN3D vs Steady and Freq Correction. 50% Span, $M = 0.8$, $\text{Freq.} = 15 \text{ Hz}$

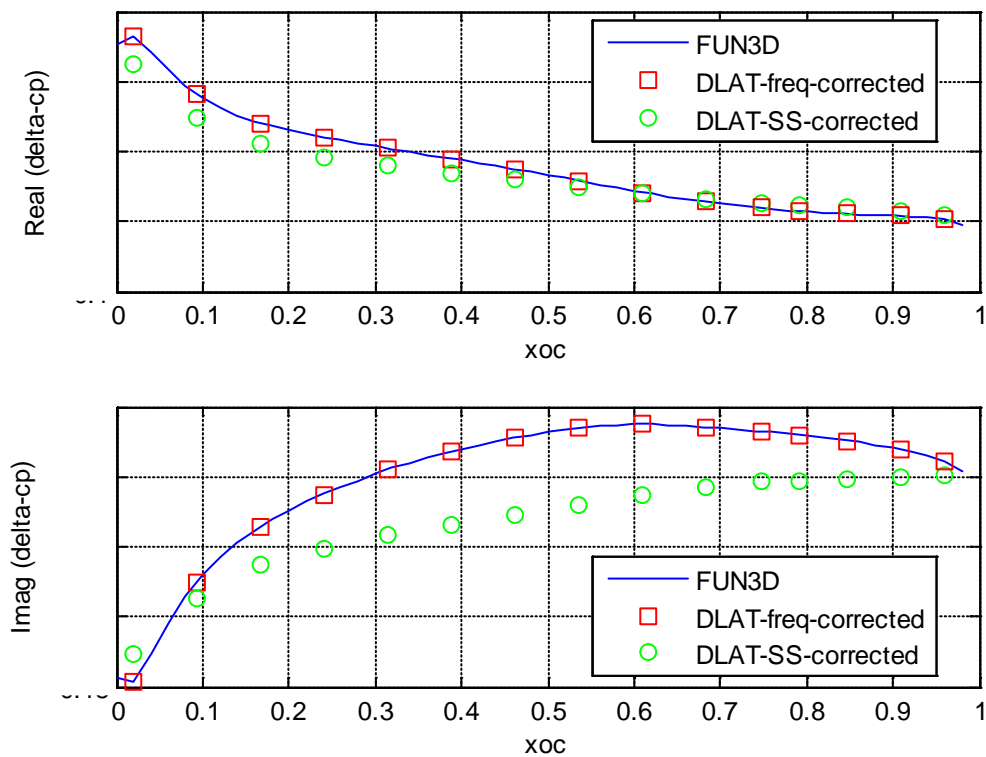


Figure 18. FUN3D vs Steady and Freq Correction. 50% Span, $M = 0.8$, Freq.=20 Hz

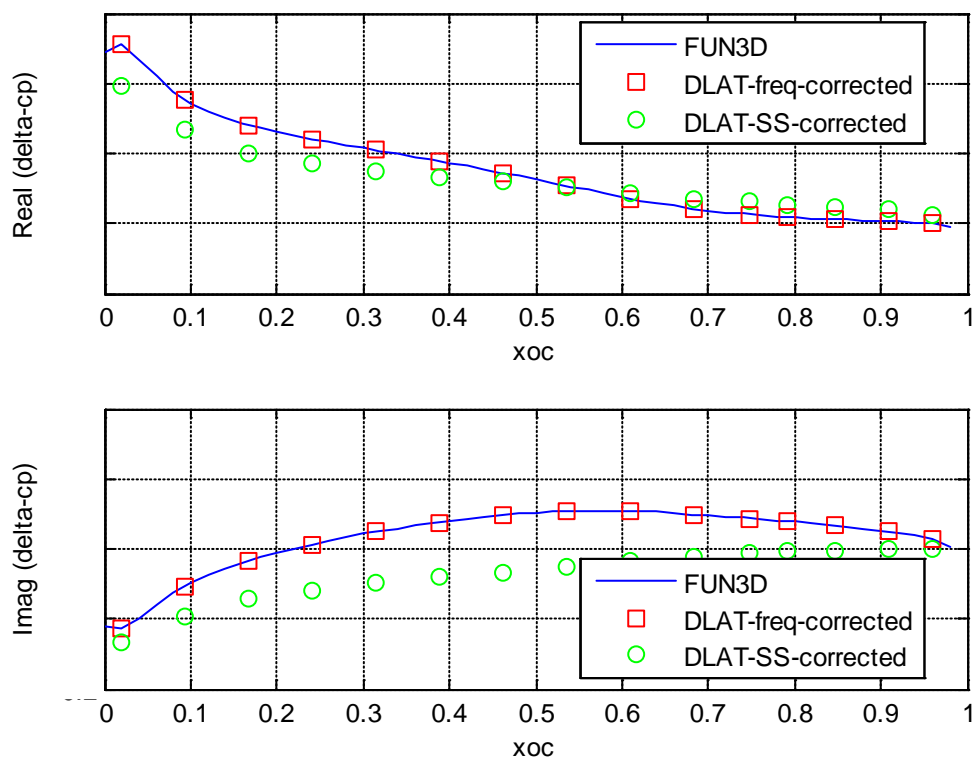


Figure 19. FUN3D vs Steady and Freq Correction. 50% Span, $M = 0.8$, Freq.=25 Hz

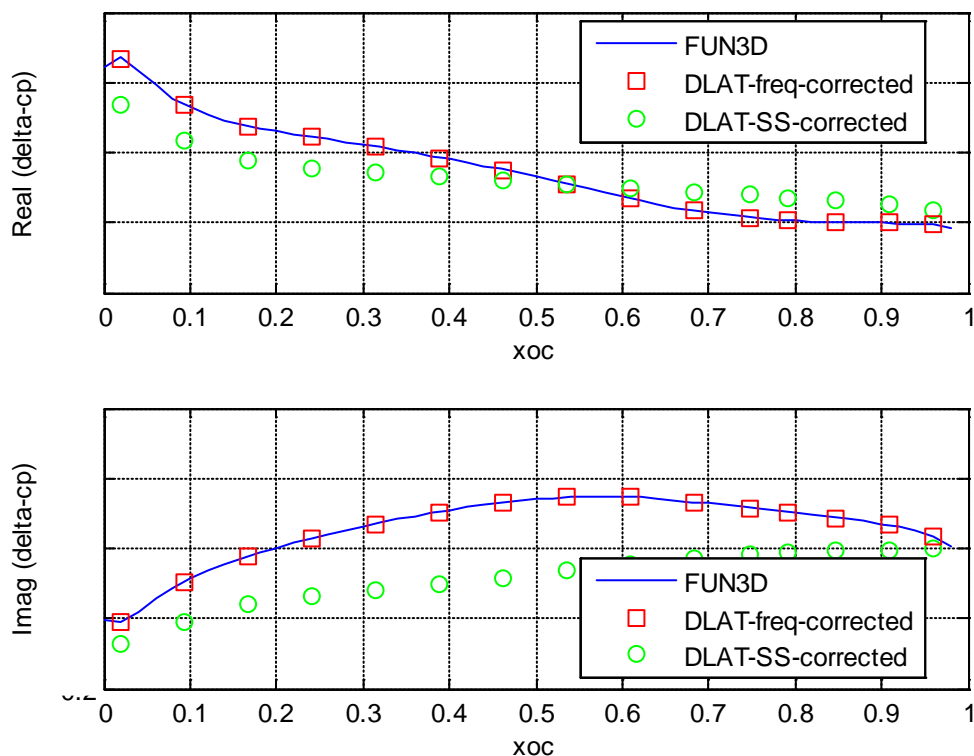


Figure 20. FUN3D vs Steady and Freq Correction. 50% Span, $M=0.8$, $\text{Freq.}=31.7$ Hz

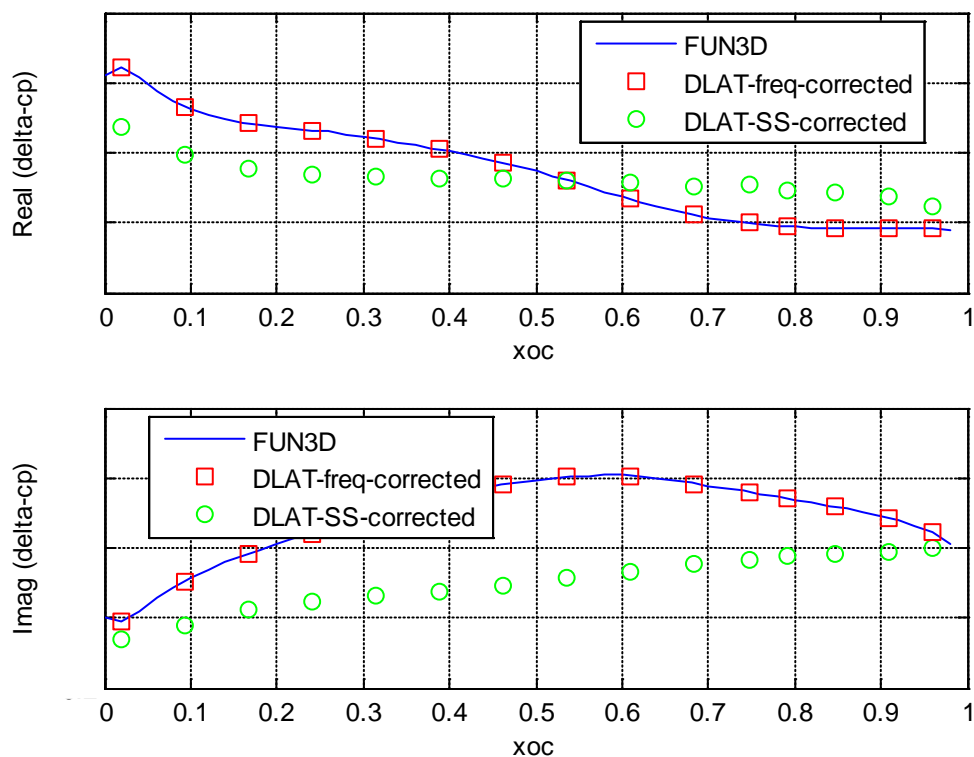


Figure 21. FUN3D vs Steady and Freq Correction. 50% Span, $M=0.8$, $\text{Freq.}=39.6$ Hz

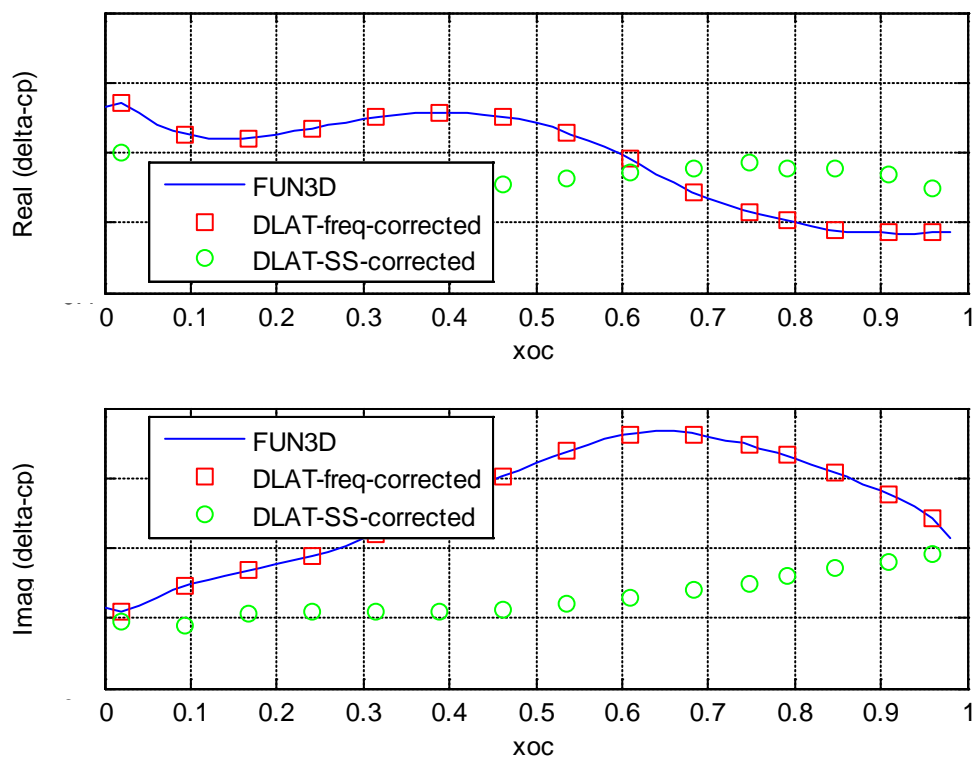


Figure 22. FUN3D vs Steady and Freq Correction. 50% Span, $M=0.8$, Freq.=59.5 Hz

VIII. Flutter Results Using Different DLM Corrections

Flutter calculations have been performed with all of the above presented steady and unsteady aero corrections, and the results are presented in Table 1.

			Flutter Point (psf)	Flutter Frequency (Hz)
M=0.8	Measured (M 0.78)		209	50.0
	Measured (M 0.83)		169	46.3
	No aero corrections		154	55.0
	Steady ECFT		153	55.1
	Unsteady ECFT		183	54.3
	Steady Diagonal		224	55.5
Unsteady Diagonal			Unrealistic results with multiple very-low qbar flutter modes	

Table 1. Flutter Results with Different Aero Corrections

From the previous table, it is significant that both the steady ECFT and Steady diagonal corrections produce exactly the same pressure distribution per unitary pitch change, but they produce flutter speeds that are quite different from each other. ECFT corrected with a steady pitching does not differ much from the results obtained

with the uncorrected DLM, but is quite conservative, whereas the diagonal steady correction clearly produces flutter speeds above the measured points.

The use of the unsteady frequency correction gives a more accurate result, within the uncertainty band of the flutter test. However, unsteady diagonal correction produces completely unrealistic results and puts into question the validity of the method.

IX. Conclusions and Future Work

The current aerodynamic correction methods have been outlined, and the results produced compared against experimental data. Both the steady and unsteady ECFT show the most promising results for use in an industrial environment.

Pure diagonal methods do not seem to produce realistic results for the mechanism considered. However, preliminary results with the modified ECFT (quasidiagonal) method are very close to the ones obtained with the unsteady ECFT, with a significantly reduced cost. This approach is currently being investigated and will be the topic of future research.

Preliminary comparison of all the methods at high Mach numbers (e.g. $M 0.95$), show very conservative results. In future work this condition will be analyzed with variations on the aerodynamic states to generate the aerodynamic corrections.

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